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Exercises

14.1. Bell measurement. Consider a system of two particles of spin $1/2$, whose spin state is written as

$$\alpha|+; +\rangle + \beta|+; -\rangle + \gamma|-\rangle + \delta|-\rangle, \quad (14.23)$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$.

- The component of the spin of each particle along the z axis is measured. What are the possible results and the corresponding probabilities?
- Rather than the preceding measurement, a detection which projects the spin state of the two particles onto one of the four states of the *Bell basis*,

$$\begin{aligned} |\Psi_+\rangle &= \frac{1}{\sqrt{2}} (|+; +\rangle + |-\rangle), & |\Phi_+\rangle &= \frac{1}{\sqrt{2}} (|+; -\rangle + |-\rangle), \\ |\Psi_-\rangle &= \frac{1}{\sqrt{2}} (|+; +\rangle - |-\rangle), & |\Phi_-\rangle &= \frac{1}{\sqrt{2}} (|+; -\rangle - |-\rangle), \end{aligned}$$

is performed. What is the probability of each of the four possible results?

14.2. Quantum teleportation of a spin state. Alice has a spin- $1/2$ particle A in the spin state

$$\alpha|+\rangle + \beta|-\rangle, \quad \text{where} \quad |\alpha|^2 + |\beta|^2 = 1,$$

that she wants to teleport to Bob. Alice and Bob also have a pair of spin- $1/2$ particles B and C , prepared in the singlet state

$$\frac{1}{\sqrt{2}} (|+; -\rangle - |-\rangle)$$

(see Fig. 14.4)

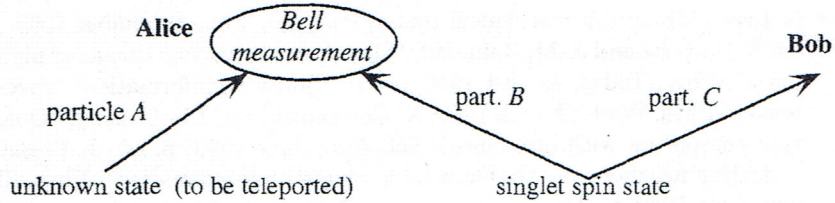


Fig. 14.4. Principle of the quantum teleportation of the quantum state of a particle

- Alice performs a measurement of the spin state of AB , which projects this state onto one of the four vectors of the Bell basis of AB (see the preceding exercise). What are the probabilities of each of the four possible results?
- Suppose that Alice finds the pair AB in the spin state $|\Phi_{-}\rangle$. What is the spin state of particle C after this measurement?
- Deduce from the preceding questions the principle of quantum teleportation.
- Can this principle be used to transmit information from Alice to Bob faster than with classical channels (and thus faster than the speed of light)?